Topological insulators

APS March Meeting tutorial, 20 March 2011

Joel Moore
University of California, Berkeley,
and Lawrence Berkeley National Laboratory
Thanks

Collaborations

Berkeley students:
Gil Young Cho
Andrew Essin (UCB→Boulder)
Roger Mong
Vasudha Shivamoggi
Cenke Xu (UCB→Harvard→UCSB)

Leon Balents, Marcel Franz, Babak Seradjeh, David Vanderbilt, Xiao-Gang Wen

Berkeley postdocs:
Jens Bardarson
Pouyan Ghaemi
Ying Ran (UCB→Boston College)
Shinsei Ryu
Ari Turner

Discussions

Berkeley:
Dung-Hai Lee, Joe Orenstein, Shinsei Ryu, R. Ramesh, Ivo Souza, Ashvin Vishwanath
Special thanks also to
Duncan Haldane, Zahid Hasan, Charles Kane, Laurens Molenkamp, Shou-Cheng Zhang

References

A. M. Essin, J. E. Moore, and D. Vanderbilt, PRL 2009.

“The birth of topological insulators”

Technical reviews by Hasan and Kane (RMP colloquium) and 3D review by Hasan and JEM.

Monday, March 28, 2011
Relationship to other materials/talks

This talk will focus on introducing the basic single-electron theory of topological insulators.

We then give some examples of how their features differ from those of similar systems like graphene and the quantum Hall effect.

Experimental context will be provided in the talks of Zahid Hasan and Yulin Chen.

Marcel Franz and Xiaoliang Qi will explain how the theory extends to interacting systems and new effects appear.

For less formal reading than the review articles, my web page (socrates.berkeley.edu/~jemoore) has lecture notes and slides from a 5-hour MIT minicourse on topological phases.
Outline

1. Are there topological phases in 2D and 3D materials in zero applied field?
   Yes — “topological insulators”
   (experimental confirmation 2007 for 2D, 2008 for 3D)
   What makes an insulator topological?

2. Features of the metallic surface states
   A. Robustness to non-magnetic disorder
   B. Magnetotransport in TIs
   C. Key surface phenomena for spintronic and other applications
Types of order

Much of condensed matter is about how different kinds of order emerge from interactions between many simple constituents.

Until 1980, all ordered phases could be understood as “symmetry breaking”:

an ordered state appears at low temperature when the system spontaneously loses one of the symmetries present at high temperature.

Examples:
Crystals break the translational and rotational symmetries of free space.
The “liquid crystal” in an LCD breaks rotational but not translational symmetry.
Magnets break time-reversal symmetry and the rotational symmetry of spin space.
Superfluids break an internal symmetry of quantum mechanics.
Types of order

At high temperature, entropy dominates and leads to a disordered state. At low temperature, energy dominates and leads to an ordered state.

In case this sounds too philosophical, there are testable results that come out of the “Landau theory” of symmetry-breaking:

\[ \rho_L - \rho_G \sim \left( \frac{T_C - T}{T_C} \right)^\beta \]

Experiment: \( \beta = 0.322 \pm 0.005 \)
Theory: \( \beta = 0.325 \pm 0.002 \)

“Universality” at continuous phase transitions (Wilson, Fisher, Kadanoff, ...)

Monday, March 28, 2011
Types of order

In 1980, the first ordered phase beyond symmetry breaking was discovered.

Electrons confined to a plane and in a strong magnetic field show, at low enough temperature, plateaus in the “Hall conductance”:

force \( I \) along \( x \) and measure \( V \) along \( y \)

on a plateau, get

\[
\sigma_{xy} = n \frac{e^2}{h}
\]

at least within \( 1 \) in \( 10^9 \) or so.

What type of order causes this precise quantization?

Note I: the AC Josephson effect between superconductors similarly allows determination of \( e/h \).

Note II: there are also fractional plateaus, about which more later.
Topological order

What type of order causes the precise quantization in the Integer Quantum Hall Effect (IQHE)?

Definition I:

In a topologically ordered phase, some physical response function is given by a “topological invariant”.

What is a topological invariant? How does this explain the observation?

Definition II:

A topological phase is insulating but always has *metallic edges/surfaces* when put next to vacuum or an ordinary phase.

What does this have to do with Definition I?

“Topological invariant” = quantity that does not change under continuous deformation

(A third definition: phase is described by a “topological field theory”)

Monday, March 28, 2011
Traditional picture: Landau levels

Normally the Hall ratio is (here $n$ is a density)

$$R_H = \frac{I_x}{V_y B} = \frac{1}{nec} \Rightarrow \sigma_{xy} = \frac{nec}{B}$$

Then the value (now $n$ is an integer)

$$\sigma_{xy} = n \frac{e^2}{h}$$

corresponds to an areal density

$$\frac{n}{2\pi \ell^2} = neB/hc.$$ 

This is exactly the density of “Landau levels”, the discrete spectrum of eigenstates of a 2D particle in an orbital magnetic field, spaced by the cyclotron energy. The only “surprise” is how precise the quantization is.
Traditional picture: Landau levels and edge states

So a large system has massively degenerate Landau levels if there is no applied potential.

\[
\sigma_{xy} = n \frac{e^2}{h}, \quad \frac{n}{2\pi \ell^2} = neB/hc.
\]

\[
E = (n + 1/2)\hbar\omega_c, \quad \omega_c = \text{cyclotron frequency}
\]

Note: for a relativistic fermion, as in graphene, \( n \) goes as \( \sqrt{B} \).

In a slowly varying applied potential, the local occupation changes; at some points Landau levels are fractionally filled and there are metallic “edge states”.

Blackboard interlude: What happens with disorder? Where is the topology? (Laughlin argument and edge vs. bulk transport)
Topological invariants

Most *topological* invariants in physics arise as integrals of some *geometric* quantity.

Consider a two-dimensional surface.

At any point on the surface, there are two radii of curvature. We define the signed “Gaussian curvature” \( \kappa = \left( \frac{r_1 r_2}{r_1 + r_2} \right)^{-1} \)

Now consider *closed* surfaces.

The area integral of the curvature over the whole surface is “quantized”, and is a topological invariant (Gauss-Bonnet theorem).

\[
\int_M \kappa \, dA = 2\pi \chi = 2\pi (2 - 2g)
\]

where the “genus” \( g = 0 \) for sphere, 1 for torus, \( n \) for “\( n \)-holed torus”.

---

*Monday, March 28, 2011*
Topological invariants

Good news:
for the invariants in the IQHE and topological insulators,
we need one fact about solids

Bloch’s theorem:
One-electron wavefunctions in a crystal
(i.e., periodic potential) can be written

\[ \psi(\mathbf{r}) = e^{i \mathbf{k} \cdot \mathbf{r}} u_{\mathbf{k}}(\mathbf{r}) \]

where \( k \) is “crystal momentum” and \( u \) is periodic (the same in every unit cell).

Crystal momentum \( k \) can be restricted to the Brillouin zone, a region of \( k \)-space with periodic boundaries.
As \( k \) changes, we map out an “energy band”. Set of all bands = “band structure”.

The Brillouin zone will play the role of the “surface” as in the previous example,

and one property of quantum mechanics, the Berry phase

which will give us the “curvature”.

Monday, March 28, 2011
Berry phase

What kind of “curvature” can exist for electrons in a solid?

Consider a quantum-mechanical system in its (nondegenerate) ground state.

The adiabatic theorem in quantum mechanics implies that, if the Hamiltonian is now changed slowly, the system remains in its time-dependent ground state.

But this is actually very incomplete (Berry).

When the Hamiltonian goes around a closed loop $k(t)$ in parameter space, there can be an irreducible phase

$$\phi = \oint \mathbf{A} \cdot d\mathbf{k}, \quad \mathbf{A} = \langle \psi_k | - i \nabla_k | \psi_k \rangle$$

relative to the initial state.

Why do we write the phase in this form? Does it depend on the choice of reference wavefunctions?
Berry phase

Why do we write the phase in this form? Does it depend on the choice of reference wavefunctions?

\[ \phi = \oint A \cdot d\mathbf{k}, \quad A = \langle \psi_k | - i \nabla_k | \psi_k \rangle \]

If the ground state is non-degenerate, then the only freedom in the choice of reference functions is a local phase:

\[ \psi_k \rightarrow e^{i \chi(k)} \psi_k \]

Under this change, the “Berry connection” \( A \) changes by a gradient,

\[ A \rightarrow A + \nabla_k \chi \]

just like the vector potential in electrodynamics.

So loop integrals of \( A \) will be gauge-invariant, as will the curl of \( A \), which we call the “Berry curvature”.

\[ \mathcal{F} = \nabla \times A \]
Berry phase in solids

In a solid, the natural parameter space is electron momentum.

The change in the electron wavefunction within the unit cell leads to a Berry connection and Berry curvature:

$$\psi(r) = e^{i \mathbf{k} \cdot \mathbf{r}} u_k(r)$$

$$\mathcal{A} = \langle u_k | -i \nabla_k | u_k \rangle \quad \mathcal{F} = \nabla \times \mathcal{A}$$

We keep finding more physical properties that are determined by these quantum geometric quantities.

The first was that the integer quantum Hall effect in a 2D crystal follows from the integral of $F$ (like Gauss-Bonnet!). Explicitly,

$$n = \sum_{bands} \frac{i}{2\pi} \int d^2k \left( \left\langle \frac{\partial u}{\partial k_1} | \frac{\partial u}{\partial k_2} \right\rangle - \left\langle \frac{\partial u}{\partial k_2} | \frac{\partial u}{\partial k_1} \right\rangle \right) \quad \mathcal{F} = \nabla \times \mathcal{A}$$

$$\sigma_{xy} = n \frac{e^2}{h} \quad \text{TKNN, 1982} \quad \text{“first Chern number”}$$
The importance of the edge

But wait a moment...

This invariant exists if we have energy bands that are either full or empty, i.e., a “band insulator”.

How does an insulator conduct charge?

Answer: (Laughlin; Halperin)

There are metallic edges at the boundaries of our 2D electronic system, where the conduction occurs.

These metallic edges are “chiral” quantum wires (one-way streets). Each wire gives one conductance quantum \( (e^2/h) \).

The topological invariant of the bulk 2D material just tells how many wires there have to be at the boundaries of the system.

How does the bulk topological invariant “force” an edge mode?
The importance of the edge

The topological invariant of the bulk 2D material just tells how many wires there have to be at the boundaries of the system.

How does the bulk topological invariant “force” an edge mode?

Answer:

Imagine a “smooth” edge where the system gradually evolves from IQHE to ordinary insulator. The topological invariant must change.

But the definition of our “topological invariant” means that, if the system remains insulating so that every band is either full or empty, the invariant cannot change.

∴ the system must not remain insulating.

(What is “knotted” are the electron wavefunctions)
2005-present and “topological insulators”

The same idea will apply in the new topological phases discovered recently:

a “topological invariant”, based on the Berry phase, leads to a nontrivial edge or surface state at any boundary to an ordinary insulator or vacuum.

However, the physical origin, dimensionality, and experiments are all different.

We discussed the IQHE so far in an unusual way. The magnetic field entered only through its effect on the Bloch wavefunctions (no Landau levels!).

This is not very natural for a magnetic field. It is ideal for spin-orbit coupling in a crystal.
Spin-orbit coupling appears in nearly every atom and solid. Consider the standard atomic expression

$$H_{SO} = \lambda \mathbf{L} \cdot \mathbf{S}$$

For a given spin, this term leads to a momentum-dependent force on the electron, somewhat like a magnetic field.

The spin-dependence means that the time-reversal symmetry of SO coupling (even) is different from a real magnetic field (odd).

It is possible to design lattice models where spin-orbit coupling has a remarkable effect: (Murakami, Nagaosa, Zhang 04; Kane, Mele 05)

spin-up and spin-down electrons are in IQHE states, with opposite “effective magnetic fields”.
The “quantum spin Hall effect”

In this type of model, electron spin is conserved, and there can be a “spin current”.

An applied electrical field causes oppositely directed Hall currents of up and down spins.

The charge current is zero, but the “spin current” is nonzero, and even quantized!

However...

1. In real solids there is no conserved direction of spin.

2. So in real solids, it was expected that “up” and “down” would always mix and the edge to disappear.

3. The theory of the above model state is just two copies of the IQHE.
The 2D topological insulator

It was shown in 2005 (Kane and Mele) that, in real solids with all spins mixed and no "spin current", something of this physics does survive.

In a material with only spin-orbit, the "Chern number" mentioned before always vanishes.

Kane and Mele found a new topological invariant in time-reversal-invariant systems of fermions.

But it isn’t an integer! It is a Chern parity ("odd" or "even"), or a "Z2 invariant".

Systems in the "odd" class are "2D topological insulators"

1. Where does this "odd-even" effect come from?
2. What is the Berry phase expression of the invariant?
3. How can this edge be seen?
The “Chern insulator” and QSHE

Haldane showed that although broken time-reversal is necessary for the QHE, it is not necessary to have a net magnetic flux.

Imagine constructing a system (“model graphene”) for which spin-up electrons feel a pseudofield along $z$, and spin-down electrons feel a pseudofield along -$z$.

Then $SU(2)$ (spin rotation symmetry) is broken, but time-reversal symmetry is not:

an edge will have (in the simplest case)
a clockwise-moving spin-up mode
and a counterclockwise-moving spin-down mode
(Murakami, Nagaosa, Zhang, ’04)
The 2D topological insulator

1. Where does this “odd-even” effect come from?

In a time-reversal-invariant system of electrons, all energy eigenstates come in degenerate pairs.

The two states in a pair cannot be mixed by any $T$-invariant perturbation. (disorder)

So an edge with a single Kramers pair of modes is perturbatively stable (C. Xu-JEM, C. Wu et al., 2006).
The 2D topological insulator

1. Where does this “odd-even” effect come from?

In a time-reversal-invariant system of electrons, all energy eigenstates come in degenerate pairs. The two states in a pair cannot be mixed by any T-invariant perturbation. (disorder)

So an edge with a single Kramers pair of modes is perturbatively stable (C. Xu-JEM, C. Wu et al., 2006).

But this rule does not protect an ordinary quantum wire with 2 Kramers pairs:

The topological vs. ordinary distinction depends on time-reversal symmetry.
The 2D topological insulator

Another way to understand Z2-ness (Kane and Mele)

1. Imagine a 2D system that has a 1D edge. Assume translation invariance along the edge.

2. There is then a 1D band structure. The bulk states form continua above and below the band gap.
The 2D topological insulator

Another way to understand Z2-ness (Kane and Mele)

1. Imagine a 2D system that has a 1D edge. Assume translation invariance along the edge.

2. There is then a 1D band structure. The bulk states form continua above and below the band gap.

3. Time-reversal imposes constraints: $\pm k$ have the same energies, and there have to be degeneracies at the time-reversal symmetric points $k=0$, $k=\pm \pi/a$.

There are two topologically different ways to connect up the T-symmetric points.

The two lines out of each Kramers point in the gap either connect to the same Kramers point (OI) or different Kramers points.
The 2D topological insulator

Another way to understand Z2-ness (Kane and Mele)

1. Imagine a 2D system that has a 1D edge. Assume translation invariance along the edge.

2. There is then a 1D band structure. The bulk states form continua above and below the band gap.

3. Time-reversal imposes constraints: ±k have the same energies, and there have to be degeneracies at the time-reversal symmetric points $k=0, k=\pm \pi/a$.

There are two topologically different ways to connect up the T-symmetric points.

The two lines out of each Kramers point in the gap either connect to the same Kramers point (OI) or different Kramers points.

What about an explicit Hamiltonian?
Example: Kane-Mele-Haldane model for graphene

The spin-independent part consists of a tight-binding term on the honeycomb lattice, plus possibly a sublattice staggering

$$H_0 = -t \sum_{\langle ij \rangle} c_{i \sigma}^\dagger c_{j \sigma} + \lambda v \sum_i \xi_i c_{i \sigma}^\dagger c_{i \sigma}$$

$$\xi_i = \begin{cases} 1 & \text{if } i \text{ in } A \text{ sublattice} \\ -1 & \text{if } i \text{ in } B \text{ sublattice} \end{cases}$$

The first term gives a semimetal with Dirac nodes (as in graphene).

The second term, which appears if the sublattices are inequivalent (e.g., BN), opens up a (spin-independent) gap.

When the Fermi level is in this gap, we have an ordinary band insulator.
Example: Kane-Mele-Haldane model for graphene

The spin-independent part consists of a tight-binding term on the honeycomb lattice, plus possibly a sublattice staggering

\[ H_0 = -t \sum_{\langle ij \rangle} c_{i\sigma}^\dagger c_{j\sigma} + \lambda_v \sum_i \xi_i c_{i\sigma}^\dagger c_{i\sigma} \]

The spin-dependent part contains two SO couplings

\[ H' = i\lambda_{SO} \sum_{\langle\langle ij \rangle\rangle} v_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{\langle ij \rangle} c_i^\dagger (s \times \hat{d}_{ij})_z c_j \]

The first spin-orbit term is the key: it involves second-neighbor hopping \((v_{ij} \text{ is } \pm 1 \text{ depending on the sites})\) and Sz. It opens a gap in the bulk and acts as the desired “pseudofield” if large enough.

\[ v_{ij} \propto (d_1 \times d_2)_z \]

Claim: the system with an SO-induced gap is fundamentally different from the system with a sublattice gap: it is in a different phase. It has gapless edge states for any edge (not just zigzag).
Example: Kane-Mele-Haldane model for graphene

\[ H_0 = -t \sum_{\langle ij \rangle} c_{i\sigma}^\dagger c_{j\sigma} + \lambda_v \sum_i \xi_i c_{i\sigma}^\dagger c_{i\sigma} \]

\[ H' = i\lambda_{SO} \sum_{\langle\langle ij \rangle\rangle} v_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{\langle ij \rangle} c_i^\dagger (s \times \hat{d}_{ij})_z c_j \]

Without Rashba term (second SO coupling), have two copies of Haldane’s IQHE model. All physics is the same as IQHE physics.

The Rashba term violates conservation of Sz--how does this change the phase? Why should it be stable once up and down spins mix?
Invariants in T-invariant systems?

If a quantum number (e.g., Sz) can be used to divide bands into “up” and “down”, then with T invariance, one can define a “spin Chern integer” that counts the number of Kramers pairs of edge modes:

\[ n^\uparrow + n^\downarrow = 0, \quad n^\uparrow - n^\downarrow = 2n_s \]
What about T-invariant systems?

If a quantum number (e.g., Sz) can be used to divide bands into “up” and “down”, then with T invariance, one can define a “spin Chern number” that counts the number of Kramers pairs of edge modes:

\[ n_\uparrow + n_\downarrow = 0, \quad n_\uparrow - n_\downarrow = 2n_s \]

For general spin-orbit coupling, there is no conserved quantity that can be used to classify bands in this way, and no integer topological invariant.

Instead, a fairly technical analysis shows

1. each pair of spin-orbit-coupled bands in 2D has a \( \mathbb{Z}_2 \) invariant (is either “even” or “odd”), essentially as an integral over half the Brillouin zone;

2. the state is given by the overall \( \mathbb{Z}_2 \) sum of occupied bands: if the sum is odd, then the system is in the “topological insulator” phase.
2. What is the Berry phase expression of the invariant? It is an integral over half the Brillouin zone,

\[ D = \frac{1}{2\pi} \left[ \oint_{\partial(EBZ)} d\mathbf{k} \cdot \mathcal{A} - \int_{EBZ} d^2k \mathcal{F} \right] \text{ mod } 2 \]

This expression simplifies considerably in the case of inversion symmetry: one can evaluate the Z2 invariant from parity indices at the four T-symmetric points in the BZ (illustrated to right).

In evolving from “ordinary insulator” to “topological insulator”, a nonmagnetic material must pass through a semimetal or metal; there are an odd number of “band inversions”.

3. How can this edge be seen?
Experimental signatures

Key physics of the edges: robust to disorder and hence good charge conductors.

The topological insulator is therefore detectable by measuring the two-terminal conductance of a finite sample: should see maximal 1D conductance.

\[ G = \frac{2e^2}{h} \]

In other words, spin transport does not have to be measured to observe the phase.

Materials recently proposed: Bi, InSb, strained Sn (3d), HgTe (2d) (Bernevig, Hughes, and Zhang, Science (2006); experiments by Molenkamp et al. (2007) see an edge, but \( G \sim 0.3 \, G_0 \)
The 2D topological insulator

Key: the topological invariant predicts the “number of quantum wires”.

While the wires are not one-way, so the Hall conductance is zero, they still contribute to the ordinary (two-terminal) conductance.  
(Theoretical prediction: Bernevig, Hughes, Zhang, Science 2006.)

There should be a low-temperature edge conductance from one spin channel at each edge:

$$G = \frac{2e^2}{h}$$

König et al., Science (2007)

This appears in (Hg,Cd)Te quantum wells as a quantum Hall-like plateau in zero magnetic field.
Review of 3D facts

The 2D conclusion is that band insulators come in two classes: ordinary insulators (with an even number of edge modes, generally 0) “topological insulators” (with an odd number of Kramers pairs of edge modes, generally 1).

What about 3D? The only 3D IQHE states are essentially layered versions of 2D states: Mathematically, there are three Chern integers: $C_{xy}$ (for $xy$ planes in the 3D Brillouin torus), $C_{yz}$, $C_{xz}$

There are similar layered versions of the topological insulator, but these are not very stable; intuitively, adding parities from different layers is not as stable as adding integers.

However, there is an unexpected 3D topological insulator state that does not have any simple quantum Hall analogue. For example, it cannot be realized in any model where up and down spins do not mix!

Build 3D from 2D

Note that only at special momenta like \( k=0 \) is the “Bloch Hamiltonian” time-reversal invariant: rather, \( k \) and \(-k\) have \( T \)-conjugate Hamiltonians. Imagine a square BZ:

\[
H(-k) = TH(k)T^{-1}
\]

In 3D, we can take the BZ to be a cube (with periodic boundary conditions):

- think about \( xy \) planes
- 2 inequivalent planes look like 2D problem

\[
k_z = \pi/a
\]

3D “strong topological insulators” go from an 2D ordinary insulator to a 2D topological insulator (or vice versa) in going from \( k_z=0 \) to \( k_z=\pm\pi/a \).

This is allowed because intermediate planes have no time-reversal constraint.
1. This fourth invariant gives a robust 3D “strong topological insulator” whose metallic surface state in the simplest case is a single “Dirac fermion” (Fu-Kane-Mele, 2007).

2. Some fairly common 3D materials might be topological insulators!

Claim:
Certain insulators will always have metallic surfaces with strongly spin-dependent structure.

How can we look at the metallic surface state of a 3D material to test this prediction?
Topological Insulators from Spin-orbit Coupling
Semiclassical picture

1D edge of Quantum Hall Effect

1D edge of “Quantum Spin Hall Effect” (Murakami, Nagaosa, Zhang ’04; Kane and Mele ’05; Bernevig et al. ’06)
Experiment: Molenkamp group ‘07

2D surface of 3D Topological Insulator (JEM and Balents, Fu-Kane-Mele, Roy, ’07)
Experiment: Hasan group ’08
Imagine carrying out a “photoelectric effect” experiment very carefully.

Measure as many properties as possible of the outgoing electron to deduce the momentum, energy, and spin it had while still in the solid.

This is “angle-resolved photoemission spectroscopy”, or ARPES.
ARPES of topological insulators

First observation by D. Hsieh et al. (Z. Hasan group), Princeton/LBL, 2008.

This is later data on Bi$_2$Se$_3$ from the same group in 2009:

The states shown are in the “energy gap” of the bulk material--in general no states would be expected, and especially not the Dirac-conical shape.
STM of topological insulators

The surface of a simple topological insulator like Bi$_2$Se$_3$ is “1/4 of graphene”: it has the Dirac cone but no valley or spin degeneracies.

Scanning tunneling microscopy image (Roushan et al., Yazdani group, 2009)

Can also put on a magnetic field (Hanaguri et al., Andrei et al.) and look at Landau level energies characteristic of Dirac fermion.
Ways to define a 3D TI

1. The spin-orbit coupling must be strong enough that a bulk metallic phase transition is passed through as the spin-orbit coupling is increased from zero.

“An odd number of bands must be inverted”

Suggests we look at heavy, small-bandgap semiconductors.

2. Compute Z2 invariants in 2 time-reversal invariant planes.

3. With inversion symmetry (Fu and Kane, 2007): the Z2 invariant reduces to the product of parity eigenvalues at the 8 points where \( k = -k \).

Definitions beyond band structure:

4. A material with an odd number of surface Dirac fermions.

5. (A material with a quantized magnetoelectric effect when its surface is gapped--Franz talk)
Stability, or Phases versus points

True quantum phases in condensed matter systems should be robust to *disorder* and *interactions*.

Examples:
The Fermi gas is robust to repulsive interactions in 2D and 3D (the “Fermi liquid”) but *not* in 1D. In 1D, conventional metallic behavior is only seen at one fine-tuned point in the space of interactions.

The Fermi gas is robust to disorder in 3D but not in 1D or 2D (*Anderson localization*): the clean system is only a point in phase space in 1D or 2D.

The IQHE is a phase robust to both disorder and interactions.

See provided slides: The QSHE and TI are stable to disorder if time-reversal remains unbroken (and interactions; cf. Franz talk).
Remark on simple generalization of IQHE topology

TKNN, 1982: the Hall conductance is related to an integral over the magnetic Brillouin zone:

\[ \sigma_{xy} = n \frac{e^2}{h} \]

\[ n = \sum_{\text{bands}} \frac{i}{2\pi} \int d^2 k \left( \left\langle \frac{\partial u}{\partial k_1} \bigg| \frac{\partial u}{\partial k_2} \right\rangle - \left\langle \frac{\partial u}{\partial k_2} \bigg| \frac{\partial u}{\partial k_1} \right\rangle \right) \]

Niu, Thouless, Wu, 1985: many-body generalization
more generally, introducing “twist angles” around the two circles of a torus and considering the (assumed unique) ground state as a function of these angles,

\[ n = \int_0^{2\pi} \int_0^{2\pi} d\theta \, d\varphi \frac{1}{2\pi i} \left| \left\langle \frac{\partial \phi_0}{\partial \varphi} \bigg| \frac{\partial \phi_0}{\partial \theta} \right\rangle - \left\langle \frac{\partial \phi_0}{\partial \theta} \bigg| \frac{\partial \phi_0}{\partial \varphi} \right\rangle \right| \]

This quantity is an integer.

For T-invariant systems, all ordinary Chern numbers are zero.
Redefining the Berry phase with disorder

Suppose that the parameters in $H$ do not have exact lattice periodicity.

Imagine adding boundary phases to a finite system, or alternately considering a “supercell”. Limit of large supercells -> disordered system.

Effect of boundary phase is to shift $k$: alternate picture of topological invariant is in terms of half the $(\Phi_1, \Phi_2)$ torus.

Can define Chern parities by pumping, analogous to Chern numbers, and study phase diagram w/disorder.
Summary of progress c. 2009

1. There are now at least 3 strong topological insulators that have been seen experimentally (Bi$_x$Sb$_{1-x}$, Bi$_2$Se$_3$, Bi$_2$Te$_3$).

2. Their metallic surfaces exist in zero field and have the predicted form.

3. These are fairly common bulk 3D materials (and also $^3$He B).

4. The temperature over which topological behavior is observed can extend up to room temperature or so.

What’s left

What is the physical effect or response that defines a topological insulator beyond single electrons? (Qi, Franz talks)

What are some differences between graphene and the 3D TI surface, resulting from the spin-momentum locking?
Spintronic applications of 3D TIs

This is a very active area on the archive, but most of what is discussed is very simple:

A charge current at one TI surface has a nonzero average spin. The same is true for a Rashba quantum well, where the two electron sheets almost cancel; in a TI there is only one sheet and the effect is much stronger.
Application II: Spintronics

- Observation of giant spin-charge coupling

**Spin-charge coupling**
Charge current = spin density
About 100 times larger than QWs
Align 10 spins/micron\(^2\) ⇒ 1 microamp

The locking of spin and momentum at a TI surface means that a charge *current* at one surface generates a spin *density*.

Similarly a charge *density* is associated with a spin *current*.

While these effects could cancel out between the top and bottom surfaces of an unbiased thin film, any asymmetry (such as electrical bias or substrate effects) leads to a net spin-charge coupling.

(O. Yazyev, JEM, S. Louie, PRL 2011)
Goals with current materials

• Currently sought: observation of giant spin-charge coupling

First-principles calculations of surface states, including reduced spin polarization (O. Yazyev, JEM, S. Louie, PRL 2011)

1. Gives numerical strength of spin-charge coupling, e.g., in “inverse spin-galvanic effect” (Garate and Franz): use TI surface current to switch an adsorbed magnetic film

2. Can bias electrically so that combination of surfaces has net spin-charge coupling
Berry phase in transport

1. There is a Berry phase effect in a TI nanowire that is similar but different from the corresponding effect in carbon nanotubes (cf. T. Ando). This has not yet been observed experimentally but efforts continue.

2. Initially, the main materials challenge was to produce genuinely insulating TIs. Considerable progress has been made (Bi$_2$Te$_2$Se/Bi$_2$TeSe$_2$). Surface state mobility $> 10,000$.

A current challenge: where is the Zeeman effect of the surface states? The effect of the Zeeman term is different than in graphene, since it does not commute with the rest of H.

It is also larger since bulk $g >= 20$. 
Puzzles in transport

For observation of the above in existing TIs, reduction of bulk residual conductivity is important and seems to be underway.

Magnetic field experiments can isolate 2D surface state features.

Puzzle 1: Stanford nanowire experiment (Yi Cui et al., Nature Materials)

sees Aharonov-Bohm (h/e) oscillations, as expected for a clean system, rather than Sharvin & Sharvin (h/2e), as expected for a

The sign is also not what is expected in the strong-disorder limit

(Bardarson, Brouwer, JEM, PRL 2010; Zhang and Vishwanath, arXiv 1005.3042).

See F. Xiu, K. L. Wang et al., Friday talk

Puzzle 1 is a flux effect. What about field measurements?
Question: is the Zeeman effect visible in surface-state magnetoconductance? Key difference between TIs and either graphene or a 2DEG. An alternate mechanism to E-dependent velocity. Or do these cancel? Cf. Seradjeh et al. on bismuth

See N. Phuan Ong invited talk, Friday

Landau Levels with Zeeman Coupling

Surface effective Hamiltonian

\[ H = v \left( \sigma^x \pi_y - \sigma^y \pi_x \right) - \frac{g \mu_B}{2} \mathbf{B} \cdot \mathbf{\sigma} \quad (\pi = \mathbf{p} + eA) \]

Landau Levels

\[ E_{\pm n} = \begin{cases} \frac{g \mu_B}{2} |B_z| & n = 0 \\ \pm \sqrt{2n \hbar \nu^2 e|B_z| + \left( \frac{g \mu_B}{2} |B_z| \right)^2} & n > 0 \end{cases} \]
A consequence of surface “half-metal”

It turns out that the core of a magnetic vortex in a two-dimensional “$p+ip$” superconductor can have a Majorana fermion. (But we haven’t found one yet.)

However, a superconducting layer with this property exists at the boundary between a 3D topological insulator and an ordinary 3D superconductor (Fu and Kane, 2007).

Introductory talk on Majoranas in condensed matter:
JEM, Wednesday 7:30 pm, Physics “Trends” session
Some challenges:

Materials: Going to Bi$_2$Te$_2$Se is found to alleviate the residual conductivity problems of Bi$_2$Se$_3$ and Bi$_2$Te$_3$. The surface mobility is approaching levels sufficient for FQHE behavior.

Adding copper to Bi$_2$Se$_3$ produces a (topological?) superconductor with T$_c$ around 4 K

Probes: Still no observation of predicted spin effects in transport.

Theory:
1. What is the equivalent of Landau-Ginzburg theory for TIs?

2. Are there “fractional” topological insulators that are not adiabatically connected to the noninteracting ones?

(2D: Levin and Stern; Partons: Qi et al; Swingle et al.; BF theory; Cho and JEM)
Thanks

Collaborations

Berkeley students:
Gil Young Cho
Andrew Essin (UCB→Boulder)
Roger Mong
Vasudha Shivamoggi
Cenke Xu (UCB→Harvard→UCSB)

Berkeley postdocs:
Jens Bardarson
Pouyan Ghaemi
Ying Ran (UCB→Boston College)
Shinsei Ryu
Ari Turner

Leon Balents, Marcel Franz, Babak Seradjeh, David Vanderbilt, Xiao-Gang Wen

Discussions

Berkeley:
Dung-Hai Lee, Joe Orenstein, Shinsei Ryu, R. Ramesh, Ivo Souza, Ashvin Vishwanath
Special thanks also to
Duncan Haldane, Zahid Hasan, Charles Kane, Laurens Molenkamp, Shou-Cheng Zhang

References

A. M. Essin, J. E. Moore, and D. Vanderbilt, PRL 2009.

“The birth of topological insulators”

Technical reviews by Hasan and Kane (RMP colloquium) and 3D review by Hasan and JEM.

Monday, March 28, 2011